

Is the transition redshift a new cosmological number?

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Observations from Supernovae Type Ia (SNe Ia) provided strong evidence for an expanding accelerating Universe at intermediate redshifts. This means that the Universe underwent a dynamic phase transition from deceleration to acceleration at a transition redshift z_t of the order unity whose value in principle depends on the cosmology as well as on the assumed gravitational theory. Since cosmological accelerating models endowed with a transition redshift are extremely degenerated, in principle, it is interesting to know whether the value of z_t itself can be observationally used as a new cosmic discriminator. After a brief discussion of the potential dynamic role played by the transition redshift, it is argued that future observations combining SNe Ia, the line-of-sight (or “radial”) baryon acoustic oscillations, the differential age of galaxies, as well as the redshift drift of the spectral lines may tightly constrain z_t , thereby helping to narrow the parameter space for the most realistic models describing the accelerating Universe.

I. INTRODUCTION

The extension of the Hubble diagram to larger distances by using observations from supernovae type Ia (SNe Ia) as standard candles allowed the history of cosmic expansion to be probed at much higher accuracy at low and intermediate redshifts. Independent measurements by various groups indicated that the current expansion is in fact speeding up and not slowing down, as was believed for many decades [1–3]. In other words, by virtue of some unknown mechanism, the expansion of the Universe underwent a “dynamic phase transition” whose effect was to change the sign of the universal deceleration parameter $q(z)$.

The correct physical explanation for such a transition is the most profound challenge for cosmology today. Within the General Relativistic (GR) paradigm, the simplest manner for explaining such a phenomenon is by postulating a cosmological constant Λ in the Einstein equations. Indeed, anything which contributes to a decoupled vacuum energy density also behaves like a cosmological constant. However, the existence of the so-called cosmological constant and coincidence problems [4], inspired many authors to consider alternative candidates, thereby postulating the existence of an exotic fluid with negative pressure (in addition to cold dark matter), usually called dark energy [5].

Possible theoretical explanations for the present accelerating stage without dark energy are also surprisingly abundant [6, 7]. Even in the framework of GR there are some alternative proposals where the existence of a new dark component is not necessary in order to have an ac-

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celerating regime at low redshifts. For instance, many authors have claimed that the fact that the acceleration comes out very close to the beginning of the nonlinear evolution of the contrast density is not just a trivial coincidence [8]. In this connection, several averaging procedures have been developed in order to take into account a possible “back reaction” effect associated with the existence of inhomogeneities [9]. Another possibility still within the GR framework is that ‘dynamic transition’ can be powered uniquely by the gravitationally-induced creation of cold dark matter particles [10, 11]. The basic idea is that the irreversible process of cosmological particle creation at the expense of the gravitational field can phenomenologically be described by a negative pressure and the associated entropy production [12]. Another possibility is provided by models with interaction in the dark sector, as happens, for instance, in decaying vacuum cosmologies [13], as well as, in many variants of coupled dark energy models [14].

In this paper we advocate a different approach based on the simple existence of a transition redshift (z_t) as required by the SNe Ia data. Its leitmotiv is summarized in the caption of Figure 1. In our view, due to the recent advances of astronomical observations such a quantity defining the transition between a decelerating to an accelerating stage will become a powerful cosmological probe. In particular, it is argued that future observations combining SNe Ia, the line-of-sight (or “radial”) Baryon Acoustic Oscillations (BAO), the differential ages of galaxies (DAG), as well as the redshift drift of the spectral lines (RDSL) will constrain z_t , thereby helping to narrow the parameter space for the most realistic models describing the accelerating Universe.

The article is structured as follows. In Sect. II, we discuss the general problem of the transition redshift for different cosmologies. In Sect. III, we discuss how it

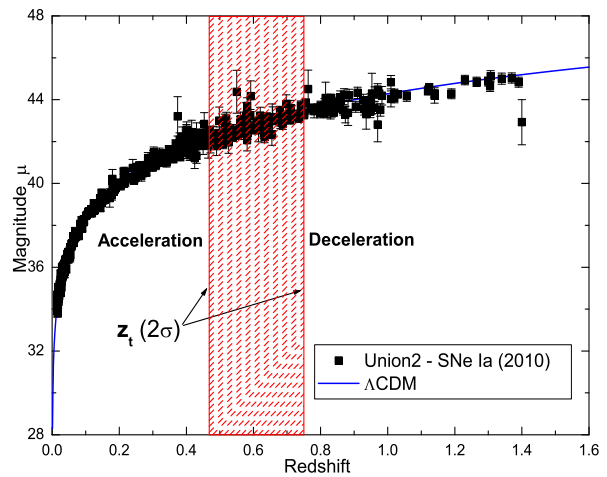


FIG. 1: The relative magnitude as a function of the redshift for the SNe Ia sample compiled by Amanullah *et al.* [3]. The vertical strip shows the Riess *et al.* [15] limits on the transition redshift, $z_t = 0.426^{+0.27}_{-0.089}$ (at 95% c.l.), based on a kinematic approach [16–19]. It will be argued here that the transition redshift will become accessible by future observations, and, as such, it may play the role of a primary cosmological parameter.

can be thought as a new cosmic parameter even in the context of the Λ CDM model. In section IV, the transition redshift is discussed as a new cosmological number in the sense of Sandage. The possibility to access it from independent observations is also discussed in the corresponding subsections. Finally, in the conclusion section we summarize the basic results.

II. TRANSITION REDSHIFT IN FRW GEOMETRIES

In what follows we restrict our attention to the class of spacetimes described by the FRW line element (unless explicitly stated we set $c = 1$):

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \quad (1)$$

where $a(t)$ is the scale factor and k is the curvature constant, which can be -1 , 0 , or 1 , for a spatially open, flat

or closed Universe, respectively. Although inflationary models and recent observations from CMB favor a spatially flat Universe, we shall not restrict ourselves to this case.

In this background, the Einstein Field Equations (EFE) and the decelerating parameter, q , can be written as:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho_T, \quad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_T + 3p_T), \quad (3)$$

$$q(z) \equiv -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{H^2} \left(\frac{\ddot{a}}{a}\right), \quad (4)$$

where ρ_T and p_T are the total energy density and pressure of the mixture and $H(t) = \dot{a}/a$ is the Hubble parameter. Note that the acceleration equation (3) does not depend explicitly on the curvature, and, similarly, the same happens with the transition redshift (z_t) since it is implicitly defined by the condition $q(z_t) = \ddot{a}(z_t) = 0$. It is also worth noticing that all kinematic approaches developed in the literature [16–19] point to a transition redshift in the past, that is, at intermediate redshifts ($z_t < 1$). The importance of such a result comes from the fact that it is independent of any dark energy models as well as of the underlying gravity theory.

As widely known, the first SNe Ia analyses were done assuming a constant Λ for the dark energy component. However, due to the coincidence and cosmological constant problems several candidates for dark energy were proposed in the literature. At present, beyond the cosmological constant there is a plethora of relativistic dark energy candidates capable to explain the late time accelerating stage, and, as such, the space parameter of the basic observational quantities is rather degenerate. The most economical explanation is provided by the flat

Λ CDM model which has only one dynamic free parameter, namely, the vacuum energy density. It seems to be consistent with all the available observations provided that the vacuum energy density is fine tuned to fit the data ($\Omega_\Lambda \sim 0.7$). However, even considering that the addition of extra fields explain the late time accelerating stage and other complementary observations [20–23], the need of (yet to be observed) dark energy component with unusual properties is certainly a severe hindrance.

For the sake of simplicity, next section we focus our attention on the Λ CDM model and its predicted transition redshift. The main aim is to show how to build a complementary space parameter based on the transition redshift as a basic quantity. Further, it will be discussed how such an approach may be useful to discriminate the realistic accelerating world models proposed in the literature.

III. TRANSITION REDSHIFT IN Λ CDM MODELS

The late time observed Universe in Λ CDM models is composed almost completely of pressureless matter (consisting of dark and normal baryonic matter components), and a negative pressure cosmological constant energy density, since the radiation contribution at low redshifts is just $\sim 10^{-5}$ of the total energy density. Following standard lines, we write deceleration parameter as

$$q(z) \equiv -\frac{1}{H^2} \left(\frac{\ddot{a}}{a}\right) = \frac{(1+z)}{H(z)} \frac{dH(z)}{dz} - 1, \quad (5)$$

with the Hubble parameter assuming the form below

$$H(z) = H_0 [\Omega_M(1+z)^3 + \Omega_\Lambda + \Omega_k(1+z)^2]^{1/2}, \quad (6)$$

where Ω_M , Ω_Λ and $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$, are the present day matter, vacuum and curvature parameters, respectively.

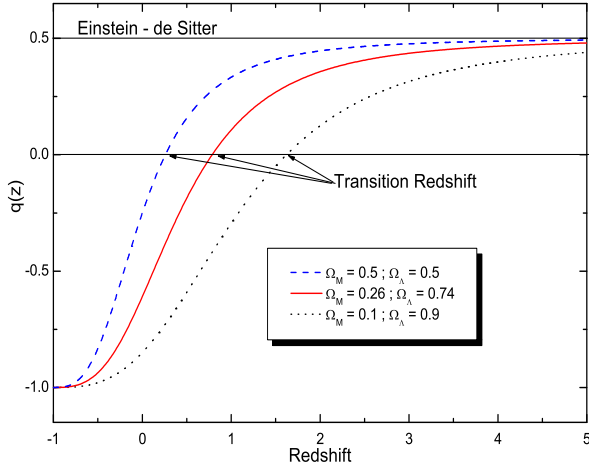


FIG. 2: Deceleration parameter as a function of the redshift for a flat Λ CDM model and some selected values of Ω_M . The solid (red) curve is the evolution of $q(z)$ for the so-called cosmic concordance model. The transition redshift is heavily dependent on the possible values of the density parameter, and, as expected, z_t is higher for smaller values of Ω_m .

In this framework, the acceleration equation (3) yields

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} [\rho_M(1+z)^3 - 2\rho_\Lambda], \quad (7)$$

while the transition redshift (where \ddot{a} vanishes) can be written as

$$z_t = \left[\frac{2\rho_\Lambda}{\rho_M} \right]^{\frac{1}{3}} - 1 = \left[\frac{2\Omega_\Lambda}{\Omega_M} \right]^{\frac{1}{3}} - 1. \quad (8)$$

As should be expected, the transition redshift does not depend explicitly on the curvature, only on the ratio of vacuum density and matter density. However, if we assume a spatially flat Universe, we obtain the normalization condition $\Omega_M + \Omega_\Lambda = 1$ and Eq. (8) becomes

$$z_t = \left[\frac{2(1 - \Omega_M)}{\Omega_M} \right]^{\frac{1}{3}} - 1. \quad (9)$$

In Figure 2, we show the deceleration parameter of a spatially flat Λ CDM model as a function of the redshift for some selected values of the matter density parameter. Note that the possible values of z_t are strongly dependent on the values assumed for Ω_M .

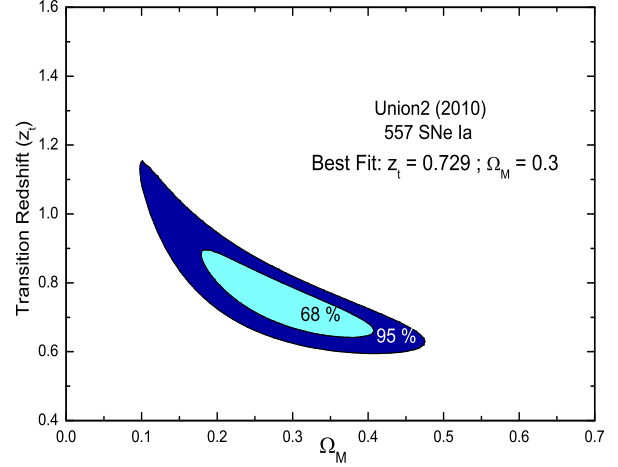


FIG. 3: Transition redshift as a function of Ω_M for a general Λ CDM model ($\Omega_k \neq 0$). The inner and outer contours (68%, 95%) C. L. show the limits on the transition redshift based on the SNe Ia sample compiled by Amanullah *et al.* [3]. The best fit values are explicitly shown in the plot.

But how can we build a suitable space parameter having the transition redshift as a basic variable? In principle, even before to discuss the related observations, we observe that such a bidimensional parameter space, say, (Ω_M, z_t) can be defined through the transition redshift itself. In the framework of a general Λ CDM model, one may combine Eqs. (8) and (6) in order to obtain an expression for $H(z_t, \Omega_M)$.

In Figure 3, we display our χ^2 -statistical analysis in the plane (Ω_M, z_t) based on the supernova sample (Union2) as compiled by Amanullah and collaborators [3]. Note that for a general Λ CDM model, the transition redshift is well constrained at 2σ confidence level. More precisely, we have found a transition redshift on the interval $0.60 \leq z_t \leq 1.18$ (2σ , joint analysis). Indirectly, this result shows that the cosmic expansion history can also be rediscussed in terms of the transition redshift.

In Figure 4, we have plotted the dependence of z_t as a function of the ratio, $r = \Omega_M/\Omega_\Lambda$, for a general Λ CDM

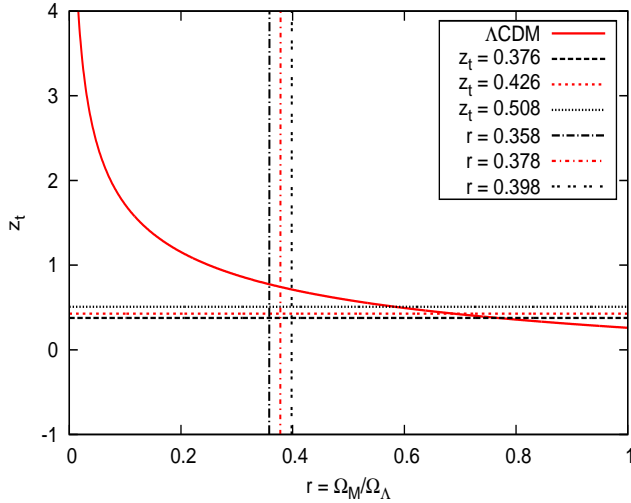


FIG. 4: Transition redshift as a function of the density ratio $r = \Omega_M/\Omega_\Lambda$. Also shown are the Riess et al. [15] 68% c.l. limits on the transition redshift, $z_t = 0.426^{+0.082}_{-0.050}$ and the derived WMAP5 68% c.l. on the ratio, $r = 0.387 \pm 0.020$.

model. The horizontal and vertical strips correspond, respectively, to $z_t = 0.426^{+0.27}_{-0.089}$ (2σ) from Riess et al. [15] and the derived WMAP5 68% c.l. on the ratio, $r = 0.387 \pm 0.020$.

In Figure (5) we display the transition redshift for the flat case as a function of the matter density parameter. As expected, in the limit $\Omega_M \rightarrow 1$ (Einstein-de Sitter model) there is no transition. The horizontal lines in both plots are the kinematic limits on z_t derived by Riess et al. [15], $z_t = 0.426^{+0.27}_{-0.089}$ (2σ), by using a linear parametrization of the deceleration parameter $q(z)$ [16]. More recently, one of us have checked their analysis [17] and have found $z_t = 0.426^{+0.082+0.27}_{-0.050-0.089}$, at 68% and 95% c.l., respectively, consistent with their result. For other parameterized deceleration parameter appeared in Ref. [19], the transition redshift is constrained with $z_t = 0.69^{+0.23}_{-0.12}$ and $z_t = 0.69^{+0.20}_{-0.12}$. The interest of such an approach is that it holds regardless of the gravity theory. The vertical lines represent the constraints derived by the WMAP7 team

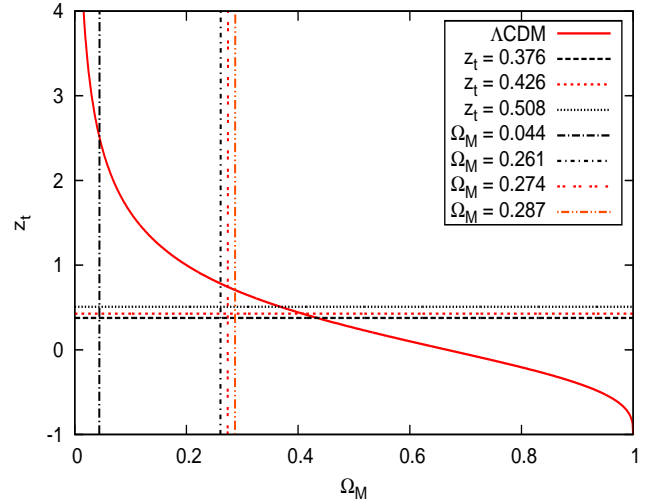


FIG. 5: Transition redshift as a function of the matter density parameter for a spatially flat Universe. Also shown are the Riess et al. [15] 68% c.l. limits on the transition redshift, $z_t = 0.426^{+0.082}_{-0.050}$ and the derived WMAP7 68% c.l. on the matter density parameter, $\Omega_M = 0.274 \pm 0.013$.

[24] through a joint analysis involving CMB, BAO and H_0 (Table 14, RECFast version 1.5 [24]). The limits at 1σ c.l. for the density parameter and density ratio are $\Omega_M = 0.274 \pm 0.013$, $r \equiv \Omega_M/\Omega_\Lambda = 0.387 \pm 0.020$, respectively.

As one may see from these figures, the standard concordance flat Λ CDM model is just marginally consistent with the transition redshift derived from the kinematic approach of Riess et al. [15]. A fortiori, this could be seen to raise some mild flags with the standard Λ CDM model, to add to the more well-known cosmological constant problem (CCP) and coincidence problem.

Thus, we can see the importance of the transition redshift in order to distinguish among similar dark energy models and as consistency check for any new dark energy model. One could rule out, for example, cosmological models with no transition redshift at all, as the family of dust filled FRW type models, and some subclasses of

$\Lambda(t)$ CDM models [13], or more generally some coupled dark matter-dark energy models.

IV. TRANSITION REDSHIFT AS A NEW COSMIC DISCRIMINATOR

In the early seventies, Alan Sandage [25] defined Cosmology as the search for two numbers: H_0 and q_0 . In the conclusions of the paper, by commenting about future observational values of H_0 and q_0 , he wrote: “The present discussion is only a prelude to the coming decade. If work now in progress is successful, better values for both H_0 and q_0 (and perhaps even Λ) should be found, and the 30-year dream of choosing between world models on the basis of kinematics alone might possibly be realized”. Indeed, it was needed to wait for almost 3 decades to obtain such quantities with great precision by using SNe Ia as standard candles [2]. Fortunately, Sandage lived enough to see his predictions substantiated by the new observational techniques.

Now, based on the results about the transition redshift presented in the earlier sections (see Figs. 1 and 2), let us discuss (from a more observational viewpoint) the possibility to enlarge Sandage’s vision by including the transition redshift, z_t , as the third cosmological number. To begin with, let us observe that the general expression for $q(z)$ as given by Eq. (5) means that the transition redshift can empirically be defined as:

$$z_t = \left[\frac{d \ln H(z)}{dz} \right]_{|z=z_t}^{-1} - 1. \quad (10)$$

Therefore, in order to access the value of z_t , a determination of $H(z)$ at least around a redshift interval involving the transition redshift becomes necessary.

How can this be worked out? In what follows, we suggest that at least 3 different kinds of ongoing and future observations can be used to obtain $H(z)$, and, therefore,

the transition redshift, namely: (i) the line-of-sight (or “radial”) baryon acoustic oscillations (BAO), (ii) the differential ages of galaxies (DAG), and (iii) the redshift drift of the spectral lines (RDSL). As we shall see, potentially, all these techniques (together or separately) are able to provide the value of z_t from the related $H(z)$ measurements. Still more important, the accuracy on the measurements of $H(z)$ must increase thereby allowing the observers to determine a value of z_t that could be useful as a robust cosmic discriminator in the near future.

A. Radial BAO

BAO in the last scattering surface provide statistical standard rulers in the late time cosmic structures of known physical lengths thereby making such measurements important cosmic probes. The first measurement of the BAO acoustic peak was obtained by Eisenstein et al. [22] through a spherical averaged two-point correlation function from luminous red galaxies data compiled by the Sloan Digital Sky Survey (SDSS). This first measurement was an average between the so called transversal (or angular) BAO, σ_{BAO} , measured in the plane of the sky, and the radial BAO, π_{BAO} , which is measured along the line of sight. Their importance as new standard rulers have also been confirmed by many independent studies [23, 28, 29]. However, the transversal BAO which provides a direct information on the angular distance is not particularly useful to determine the $H(z)$ function, and, consequently, the transition redshift (z_t).

On the other hand, the line-of-sight BAO yields an indirect measurement of $H(z)$ because it is related with the expansion history by the expression:

$$\pi_{BAO} = \frac{c \Delta z}{H(z)}, \quad (11)$$

and, hence, the values of $H(z)$ can be inferred because

π_{BAO} and Δz are observationally determined. As a matter of fact, determinations of the radial BAO have already been used to extract some $H(z)$ values at low redshift [28]. However, the situation can even be improved with the planned operation of instruments dedicated to BAO measurements. In principle, a BAO survey require redshift accuracy and coverage of enough volume (and area). In this way, an ideal instrument is needed to have a large mirror size allied to a capability for producing simultaneously a large number of spectra, for instance, through a Wide Field Multi-Object Spectrograph. There are several possibilities somewhat related with the late stages of the DFTE report (for a detailed review see Basset and Hlozek [29]).

An alternative possibility is the so-called PAU survey [30] (now called JPAS [31]) based on photometric instead of spectroscopic redshifts. The basic idea is that photometric redshifts of galaxies with enough precision to measure BAO along the line of sight may become available even with a 2.5m telescope. Their proponents claim that the survey will produce a unique data set in the optical wavelength for all objects in the north sky up to $m_B = 23 - 23.5 \text{ arcsec}^{-2}$ (5σ) thereby making the JPAS very competitive in comparison with other (photometric or spectroscopic) ground-based BAO surveys [30].

Summing up, one may expect that radial BAO measurements from luminous red galaxies and other objects with different techniques will be able to provide the instantaneous expansion rate $H(z)$ at intermediate redshifts, and, as argued here (see Eq. (10)), the transition redshift itself.

B. Differential Ages of Galaxies (DAG)

In terms of the redshift, it is easy to show that the Hubble parameter, $H(z)$, can be expressed as:

$$H(z) = -\frac{1}{(1+z)} \frac{dz}{dt}. \quad (12)$$

This simple expression means that measurements of the differential redshift ages for a class of objects, dz/dt , potentially, provide a direct estimate of $H(z)$, and, therefore, is also an interesting observational window to access the transition redshift. As recently discussed by several authors, age differences between two passively evolving galaxies formed at the same time but separated by a small redshift interval have already been inferred. In principle, the statistical significance of the measurement can be improved by selecting fair samples of passively evolving galaxies at the corresponding redshifts, and by comparing the upper cutoff in their age distributions.

In a point of fact, by choosing carefully a sample of old elliptical galaxies (similar metallicities and low star formation rates), Jimenez and collaborators used this method to obtain the first determination of the curve $H(z)$ [32]. At present, only eleven $H(z)$ data have been inferred with such a technique, some of them at high redshifts (up to $z = 1.75$). In this line, Stern et al. [33] also compiled an expanded set of $H(z)$ data (see their Figure 13) and combined it with CMB data in order to constrain dark energy parameters and the spatial curvature. More recently, simulations using Monte Carlo Technique based on Λ CDM model have also been discussed by several authors in order to constrain cosmological parameters [34]. Therefore, similarly to what happens with BAO measurements, it will be possible in the near future to obtain the expansion rate history $H(z)$ from DAG with percent precision that will translate into measurement of z_t with few times the same precision.

C. Redshift Drift

Some decades ago, Sandage [35] proposed that the dynamical expansion history could directly be traced by the time evolution of redshift (now usually called redshift drift, \dot{z}). The expansion of the Universe is expressed by the scale factor, $a(t(z)) = (1+z)^{-1}$. Therefore, the time evolution of the scale factor, or change in redshift, \dot{z} , directly measures the expansion rate of the Universe. In other words, neglecting effects of peculiar velocities, the redshift of a comoving object is a function of the observing time, and, as such, its value in the future will be different of what is measured today. As shown long ago by McVittie [36], it can be expressed as

$$\dot{z}(z) = \frac{dz}{dt_0}(t_0) = (1+z)H_0 - H(z), \quad (13)$$

where t_0 denotes the present day observing time. This clearly shows that measurements of \dot{z} are able to provide the Hubble parameter at redshift z . This redshift drift signal, \dot{z} , is indeed very small and barely accessible from techniques available at Sandage's time. It should be stressed that the redshift drift is a direct and fully model independent measurement of $H(z)$ since apart the FRW metric it does not require any cosmological assumption. The significance of this tool has been recently rediscussed by several authors by taking into account the current (and near future) observational capabilities [37–40]. By defining the apparent velocity shift, $\dot{V} = \Delta v / \Delta t_0$, it is easy to check that the redshift drift can also be rewritten as (see, for instance, Refs. [39, 40])

$$\frac{\dot{V}(z)}{c} = \frac{(1+z)H_0 - H(z)}{1+z}. \quad (14)$$

In order to use properly the above results, the first task is to identify the classes of objects and the corresponding spectral properties suitable for \dot{V} measurements. By assuming high- z observations ($z \sim 4$) for a decade, Loeb

[38] estimated $\Delta v \approx 6$ cm/s in the framework of a Λ CDM model. He claimed that such a weak signal would be measured by using the absorption lines (Ly α forest) observed in the spectra of quasi-stellar objects (QSOs).

More recently, Linske et al. [39] argued that Ly α in the redshift range ($2 \leq z \leq 5$) are indeed the most convenient targets. In their very detailed study, the possibility of Ly β forest and the influence of peculiar motions were also investigated. They also discussed the possibility of detecting and characterizing the cosmological redshift drift based on the next generation of extreme large telescope (ELT). In particular, a velocity drift experiment over 20 years using 4000 hours of observing time on a 42 meters ELT would be able to exclude $\Omega_\Lambda = 0$ with 98.1 per cent confidence level. It should be noticed that redshift drift measurements constrain $H(z)$ at high- z and, as such, would complement current and future data based on SNe Ia, radial BAO, and differential ages of galaxies.

Finally, we also observe that the idea of a direct accelerating probe (or the transition redshift) has also been discussed by Sahni, Shafieloo and Starobinsky [27] based on the expression of the average decelerating parameter \bar{q} defined by Lima [26] in connection with the total age of the Universe. By calculating the value of \bar{q} over a small redshift range close to z_t they argued that measurements of the Hubble parameter used to determine the cosmological redshift at which the universe began to accelerate, without reference to the matter density parameter. As it appears, the concept of the acceleration probe involves only the reconstruction of $H(z)$, and, therefore, it is also useful to find z_t in a model independent manner because it does not need higher derivatives of the data.

V. CONCLUSION

In this paper, we have advocated that the transition redshift should be considered as a primary cosmic parameter in the sense of Sandage [25]. By starting with a generic Λ CDM cosmology, it was shown how a convenient parameter space (Ω_M, z_t) , involving directly the transition redshift, could be built. As it appears, such an analysis might be extended for other relativistic models or even for accelerating cosmologies based on modified gravity theories. It was also argued that the transition redshift will be directly accessed through measurements of $H(z)$ by the ongoing and future observational projects. In particular, we have discussed the most promising ones

involving the line-of-sight (or “radial”) baryon acoustic oscillations, the differential age of galaxies, as well as the redshift drift of the spectral lines. Potentially, the work now in progress allied with the near future observations may transform the transition redshift in an interesting primary cosmic variable.

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